White Paper:

Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems

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1. Introduction

Broadband wireless access technologies, offering bit rates of tens of megabits per second or more to residential and business subscribers, are attractive and economical alternatives to broadband wired access technologies. Air interface standards for such broadband wireless MAN (metropolitan area networks) systems in licensed and unlicensed bands below 11 GHz are being developed by the IEEE 802.16 working group and also by the ETSI BRAN HiperMAN group. Such systems may operate over NLOS (non-line of sight) links, serving residential and SOHO (small office/home office) subscribers. In such environments multipath can be severe. This raises the question of what types of anti-multipath measures are necessary, and consistent with low cost solutions. Several variations of OFDM (Orthogonal Frequency Division Multiplexing) have been proposed as effective anti-multipath techniques, primarily because of the favourable tradeoff they offer between performance in severe multipath and signal processing complexity [Cim85], [McD96] [Sar93].

This white paper discusses an alternative approach based on more traditional single carrier modulation methods. We show that when combined with *frequency domain equalization* (FDE), this single carrier (SC) approach delivers performance similar to OFDM, with essentially the same overall complexity. In addition, single carrier modulation uses a single carrier, instead of the hundreds or thousands typically used in OFDM, so the peak-to-average transmitted power ratio for single carrier modulated signals is smaller. This in turn means that a SC system requires a smaller linear range to support a given average power, in other words, requires less peak power backoff. As such, this enables the use of cheaper power amplifier than a comparable OFDM system.

2. Multipath Channel Characteristics

[Erc99] describes multipath characteristics, based on 2 GHz measurements in suburban areas. Base antennas used in these measurements had 65º beamwidths; subscriber antennas were either omni or 32º. Up to 1 µs. delay spread[1] was observed for both omni and 32º subscriber antennas. Average delay profiles were roughly exponential, typically with a non-varying zero-delay LOS (line of sight) component.

[1] "Delay spread" in this document does not refer to rms delay spread, but rather to the total time span of the measurable channel impulse response.

[Por00] reports measured rms delay spread for MMDS systems, for 53º base beamwidths and either 10º or omnidirectional subscriber antennas. For the directional subscriber antennas and NLOS paths, about 2% of the measured paths had rms delay spreads of over 2 μs.; the average was 0.14 μs. A 2 μs. rms delay spread could be equivalent to a channel impulse response spanning roughly 8-10 μs.

[Erc01] proposes multipath channel models for use in performance evaluation of broadband wireless systems operating in the 2-11 GHz frequency range. These models address a number of different operational environments, include both LOS and NLOS channels, channel impulse response lengths up to 20 μs, and doppler frequencies of up to 2 Hz. For reference, a 5 MHz symbol rate, a 20 μs multipath delay profile spans 100 data symbols.

3. Anti-Multipath Approaches

For channel responses spanning tens or hundreds of symbols, and for wider bandwidths and higher bit rates, practical modulation alternatives are

(1) OFDM (orthogonal frequency division multiplexing);

(2) single-carrier modulation with receiver equalization done in the time domain;

(3) single-carrier modulation with receiver equalization in the frequency domain. Any of these anti-multipath approaches can be combined with antenna diversity at the transmitter and/or the receiver.

3a. OFDM (Orthogonal Frequency Division Multiplexing)

OFDM transmits multiple modulated subcarriers in parallel. Each occupies only a very narrow bandwidth. Since only the amplitude and phase of each subcarrier is affected by the channel impairments, compensation of frequency selective fading is done by compensating for each subchannel's amplitude and phase. OFDM combats multipath by performing FFT (Fast Fourier Transform) at the transmitter and receiver, with a hardware complexity (measured approximately by number of multiplications per bit) that is roughly proportional to the logarithm of maximum delay spread [Cim85]. Thus, OFDM appears to offer a better performance/complexity tradeoff than conventional single carrier modulation for large (> about 20 taps) multipath spread [McD96]. OFDM processing requires approximately $\log_2 M$ multiplies per symbol, counting both transmitter and receiver operations. A variation is adaptive OFDM, where the signal constellation on each subchannel depends on channel response at that frequency. It requires feedback from the receiver to the transmitter. It is not commonly employed in radio systems due to complexity and to channel time variations. In non-adaptive OFDM, coding and interleaving are essential to compensate for subchannels which are severely attenuated.

Because the transmitted OFDM signal is a sum of a large number (M) of slowly modulated subcarriers, it has a high peak-to-average power ratio, even if low level modulation such as QPSK is used on each subcarrier. While there are signal processing methods to reduce this ratio [Cim00], [Tar00], [Van00], the transmitter power amplifier in a OFDM system generally must be backed off several dB more than that of a single carrier system [Str01] to remain linear over the range of peaks that must be faithfully reproduced. This is especially important for subscribers near the edge of a cell, with large path loss, where BPSK or QPSK modulation must be used; the increased power backoff required in this situation for OFDM would increase the cost of the power amplifier required for such sites to 'reach' the base station. OFDM systems can also exhibit sensitivity to carrier frequency offset and phase noise [Pol95].

As previously mentioned, the full potential performance of OFDM would be reached if bit rate and power were optimized for each frequency subchannel on each base-to-subscriber link (this would require feedback of channel information from receiver to transmitter). This version of OFDM, called adaptive OFDM, is usually not done in radio systems, for reasons of complexity and and difficulty accommodating broadcast information. Some loss of performance results from the restriction to non-adaptive mode. Because of the fixed power and bit rate on each subchannel, non-adaptive OFDM must be coded, to combat frequency-selective fading. In contrast, coding is optional for single-carrier systems.

FFT signal processing in OFDM systems is done on blocks that are at least 4 to 10 times longer than the maximum impulse response span. Pilot (known) data is also often incorporated into these data blocks, for channel tracking and estimation purposes. What's more, in burst applications, one or more blocks of this size are used for initial receiver training purposes. The processing in OFDM of relatively long blocks of data at OFDM transmitters and receivers causes significantly larger total delays than in single carrier systems with time domain processing. This can be important for delay-sensitive services.

3b. Single-carrier modulation with Time Domain Adaptive Equalizer Processing

Single-carrier modulation alleviates power backoff and phase noise sensitivity problems, but the time domain complexity is proportional to delay spread in a conventional adaptive DFE (decision feedback equalizer) or linear equalizer. In general, a DFE yields better performance (lower mean squared error and bit error rate) than a linear equalizer for radio channels [Qur85], in which multipath can cause severe nulls in channel frequency response (which would cause a severe noise enhancement problem for linear equalizers). Theoretically, an equalized single-carrier system offers the same anti-multipath and anti-noise capability as adaptive OFDM [Zer89], [Sar94], [Aue98]. The minimum mean squared error (MMSE) adaptation criterion, which is relatively straightforward to implement, generally gives better performance than the zero-forcing criterion. Maximum likelihood sequence estimation (MLSE), sometimes called Viterbi equalization, is DFE, but is too complex for long impulse responses, unless the impulse response is truncated [Fal73], for example by a DFE forward filter [Mes74]. Recently, a reduced-complexity (relative to a conventional DFE) time domain adaptive DFE for long impulse responses has been developed [Ari97]. However it still requires a very long time decision feedback filter for long channel impulse responses.

3c. Single-carrier modulation with Frequency Domain Adaptive Equalizer Processing

A single carrier system transmits a single carrier, modulated at a high symbol rate. Frequency domain linear equalization in a SC system is simply the frequency domain analog of what is done by a conventional linear time domain equalizer. For channels with severe delay spread frequency domain equalization is computationally simpler than corresponding time domain equalization---for the same reason that OFDM is simpler: because equalization is performed on a block of data at a time, and the operations on this block involve an efficient FFT operation and a simple channel inversion operation. Frequency –domain equalization of single-carrier-modulated signals has been known since the early 1970's (see [Wal73], [Fer85] and references therein). Sari et al [Sar93], [Sar94], [Sar95] pointed out that when combined with FFT processing and the use of a cyclic prefix, which makes convolutions appear circular, single carrier systems with frequency domain equalization (SC-FDE) have essentially the same low complexity as OFDM systems. Frequency domain receivers processing single-carrier modulated data share a number of common signal processing functions with OFDM receivers. In fact, as we point out in

Section 4, single carrier and OFDM modems can be configured easily to coexist with one another. We also point out that there may be significant advantages to such coexistence.

Fig. 1 shows conventional linear equalization, using a transversal filter with M tap coefficients, but with filtering done in the frequency domain. The typical block length M, suitable for MMDS systems, would be in the range of 64 to 2048, for both OFDM and SC-FDE systems. There are approximately $\log_2 M$ multiplies per symbol, as in OFDM.

The use of single-carrier modulation and frequency domain equalization, by processing the FFT of the received signal [Sar94], [Sar95], [Ber95], [Kad97] has several attractive features:

- Single-carrier modulation has reduced peak-to-average ratio requirements than OFDM thereby allowing the use of less costly power amplifiers.
- Its performance with frequency domain equalization is similar to that of OFDM, even for very long channel delay spread.
- Frequency domain receiver processing has a similar complexity reduction advantage to that of OFDM: complexity is proportional to log of multipath spread.
- Coding, while desirable, is not necessary for combating frequency selectivity, as it is in non-adaptive OFDM.
- Single-carrier modulation is a well-proven technology in many existing wireless and wireline applications and its RF system linearity requirements are well known.

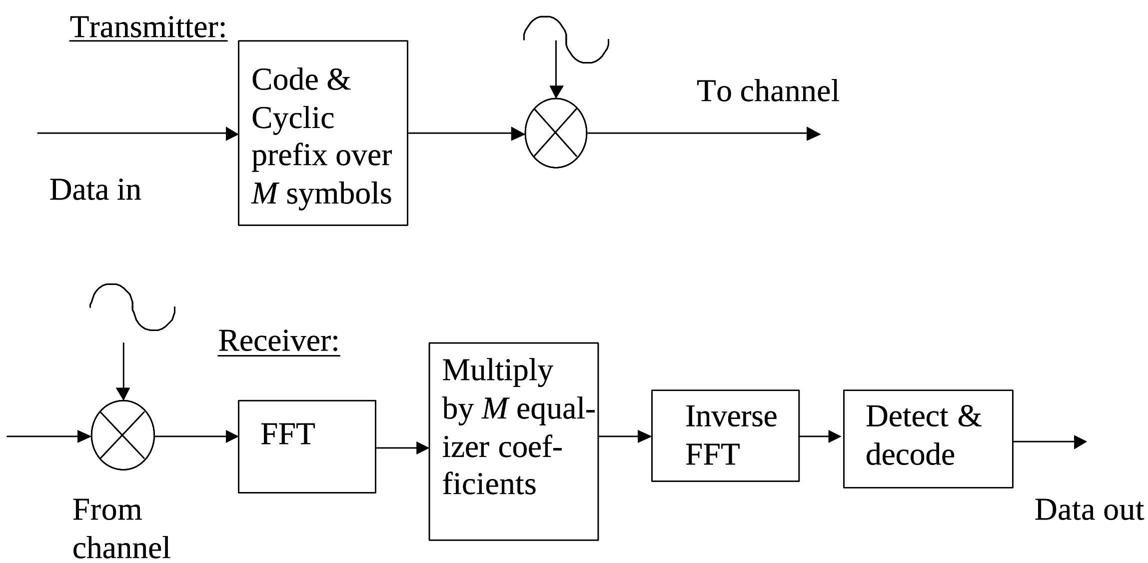


Fig. 1 SC-FDE with Linear Equalization

Usually, in both OFDM systems and in SC-FDE systems, information is transmitted in blocks, to which a *cyclic prefix* is added to remove the effect of interblock interference [Sar94], [Czy97], [Kad97], [Cla98]. The length of the cyclic prefix is the maximum expected length of the channel impulse response. As shown in Fig. 2, a cyclic prefix of length *P* symbols is formed by reproducing the sequence of the last *P* transmitted symbols in a block, and adding this sequence to the beginning of the block before transmission. At the receiver, the received cyclic prefix is discarded before processing the block. FFT processing is done on blocks of *M* symbols, the last *P* of which are used to form the cyclic prefix. The cyclic prefix transmitted at the beginning of each block has two main functions:

- It prevents contamination of a block by intersymbol interference from the previous block.
- It makes the received block appear to be <u>periodic with period *M*.</u> This produces the appearance of circular convolution, which is essential to the proper functioning of the FFT operation.

As an additional function, the cyclic prefix can be combined with a training sequence for equalizer adaptation.

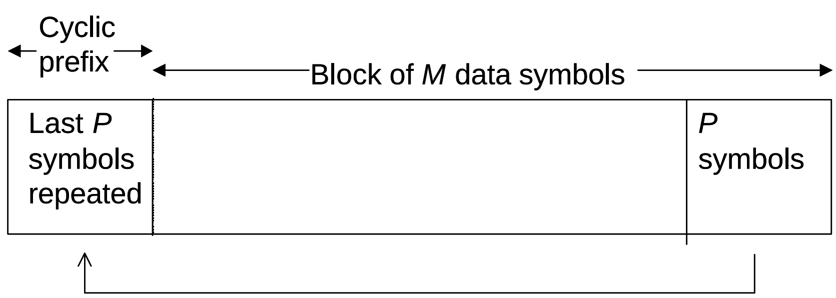


Fig. 2 Block processing in frequency domain equalization

For either OFDM or SC-FDE broadband wireless systems in severe outdoor multipath environments, typical values of *M* could be 512 or 1024, and typical values of *P* could be 64 or 128. Overlap-save or overlap-add signal processing techniques could also be used to avoid the extra overhead of the cyclic prefix [Fer85], [Hay96].

An inverse FFT returns the equalized signal to the time domain prior to the detection of data symbols [Sar94], [Kad97]. Adaptation can be done with LMS (least mean square), RLS, or least squares minimization (LS) techniques, analogous to adaptation of time domain equalizers [Hay96], [Cla98]. LS training techniques are addressed in Section 6.

Decision feedback equalization (DFE) gives better performance for frequency-selective radio channels than does linear equalization [Qur85]. In conventional DFE equalizers, symbol-by-symbol data symbol decisions are made, filtered, and immediately fed back to remove their interference effect from subsequently detected symbols. Because of the delay inherent in the block FFT signal processing, this immediate filtered decision feedback cannot be done in a frequency domain DFE, which uses frequency domain filtering of the fed-back signal. [Ber95] describes a version of a frequency domain DFE, which feeds decisions back after a certain delay. However the effect of this delay on the equalizer's performance over a wide range of radio channel responses is unclear. A hybrid time-frequency domain DFE approach, which avoids the abovementioned feedback delay problem would be to use frequency domain filtering only for the forward filter part of the DFE, and use conventional transversal filtering for the feedback part. The transversal feedback filter is relatively simple in any case, since it does not require complex multiplies, and it could be made as short or long as is required for adequate performance. Fig. 3

illustrates such a hybrid time-frequency domain DFE topology. This approach could also be used to limit possible DFE error propagation problems, or to implement MLSE equalization with a suitable truncated impulse response [Fal73], [Mes74]. Such a DFE system would train, using frequency domain processing to compute forward equalize parameters and a small matrix inversion to compute the time domain feedback taps, as described in Sections 5 and 6. Frequency domain equalization can also be combined with spatial processing to provide interference suppression and diversity [Cla98].

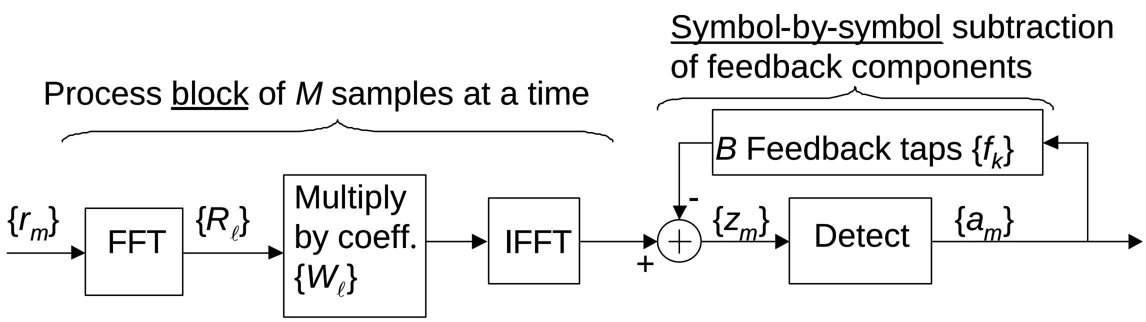
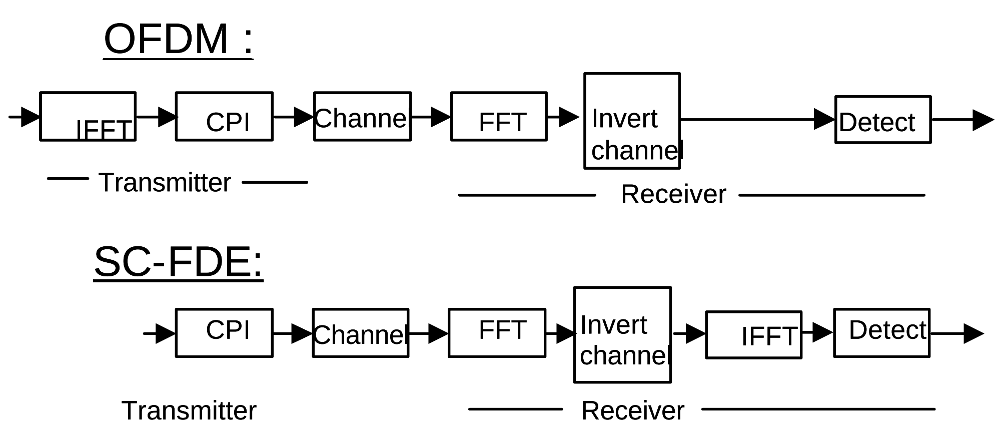


Fig. 3 SC-FDE decision feedback equalizer

4. Coexistence of Single Carrier and OFDM Systems

Fig. 4a shows block diagrams for OFDM and single carrier systems with linear frequency domain equalization. It is evident that the two types of systems differ mainly in the placement of an inverse FFT operation: in OFDM it is placed at the transmitter to multiplex the data into parallel subcarriers; in single carrier it is placed in the receiver to convert frequency domain equalized signals back into time domain symbols. The signal processing complexities of these two systems are essentially the same for equal FFT block lengths.



CPI: cyclic prefix insertion
FFT: fast Fourier transform
IFFT: inverse FFT

Fig. 4a OFDM and SC-FDE – signal processing similarities and differences

A dual-mode system, in which a software radio modem can be reconfigured to handle either single carrier or OFDM signals could be implemented by switching the inverse FFT block between the transmitter and receiver at each end of the link, as suggested in Fig. 4b.

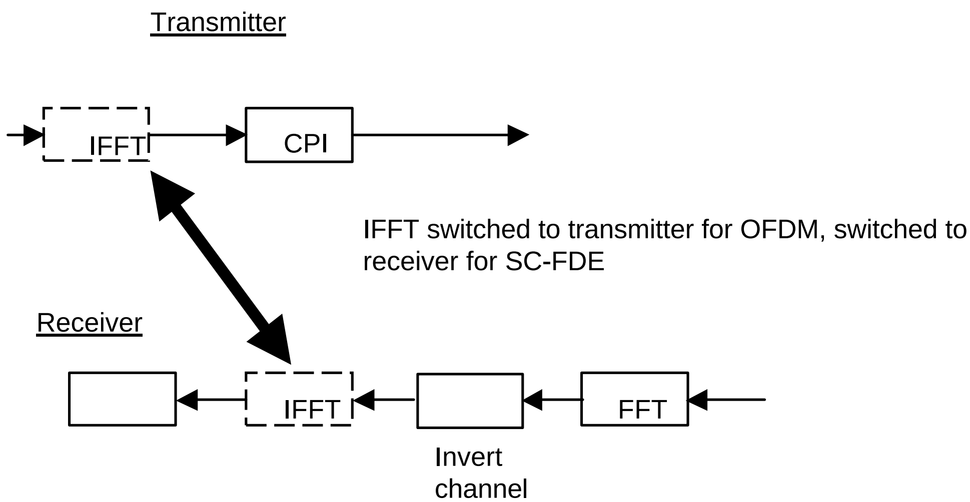


Fig. 4b Potential interoperability of SC-FDE and OFDM: a "convertible" modem

There may actually be an advantage in operating a dual mode system, wherein the base station uses a OFDM transmitter and a single carrier receiver, and the subscriber modem uses a single carrier transmitter and an OFDM receiver, as illustrated in Fig. 5. This arrangement – OFDM in the downlink and single carrier in the uplink has two potential advantages:

- Concentrating most of the signal processing complexity at the hub. The hub has two IFFT's and one FFT, while the subscriber has just one FFT.
- The subscriber transmitter is single carrier, and thus is inherently more efficient in terms of power consumption, due to the reduced power backoff requirements of the single carrier mode. This may reduce the cost of the subscriber's power amplifier.

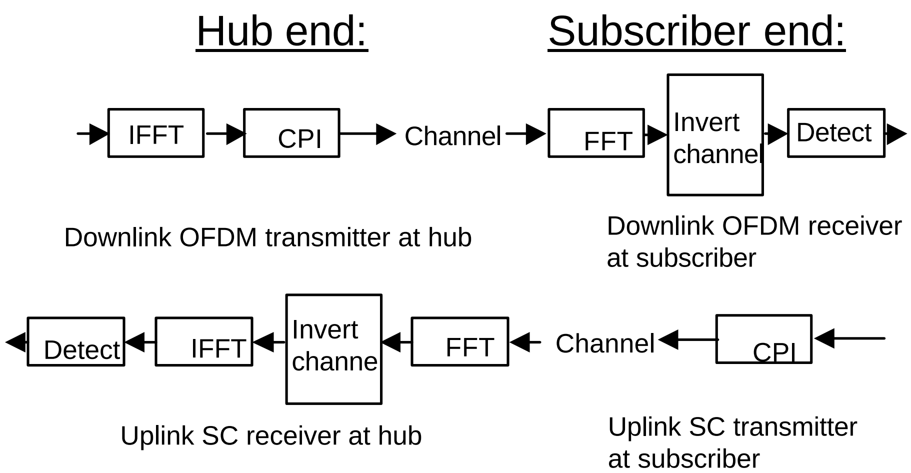


Fig. 5. Coexistence of SC-FDE and OFDM: uplink/downlink asymmetry

5. Mathematical Description of Frequency-Domain DFE (FD-DFE)

Data is transmitted in blocks of M data symbols $\{a_k\}$ at a symbol rate of $1/T$ per second. Each block is preceded by a cyclic prefix. We consider a single-carrier frequency domain DFE that processes blocks of MI received samples $\{r_m\}$ at a time, using a MI- point FFT, where I is the number of receiver input samples per data symbol, and M is the number of data symbols per FFT block. The choice of I>1 gives a fractionally-spaced equalizer whose performance is relatively insensitive to sampling phase; good performance can also be obtained for I=1 sample per symbol with an optimal sampling phase derived from a symbol timing subsystem. The data symbols are assumed to be normalized uncorrelated complex random variables derived from a discrete alphabet such as QPSK or 16QAM, with zero mean, and unit variance. The forward filter has MI complex frequency-domain coefficients $\{W_\ell\}$. After the inverse FFT operation, its time domain output is sampled once per symbol interval. There are B complex feedback coefficients $\{f_k^*\}$, $k \in F_B$, where F_B is a set of non-zero indices that correspond to the delays (in symbol periods) of the B feedback coefficients. For example, the indices F_B could correspond to the relative estimated delays of the largest channel impulse response echoes. In a number of our later examples, B=1 and F_B has just one non-zero index – the relative delay of the largest echo. For linear equalization (FD-LE) B=0, and F_B is a null set.

With this notation, we can express the mth time domain output sample, obtained by decimating the sampled forward filter output by $1/I$, as

$$z_m = \frac{1}{MI} \sum_{\ell=0}^{MI-1} W_\ell \, R_\ell \exp(j \frac{2p}{M} \ell m) - \sum_{k \in F_B} f_k^* a_{m-k}, \quad \text{where } m = 0, 1, 2, ...(M-1), \qquad (1)$$

$$\text{and where } R_\ell = \sum_{m=0}^{MI-1} r_m \exp(-j \frac{2p}{MI} \ell m), \quad \text{where } \ell = 0, 1, 2, ...(MI-1), \qquad (2)$$

is the FFT of the received *MI*-sample sequence $\{r_m\}$. Complex conjugates are denoted by asterisks. The error at the *m*th sample is

$$e_m = z_m - a_m, \tag{3}$$

and the mean squared error $E(|e_m|^2)$.) is to be minimized with respect to the $\{W_\ell\}$ and $\{f_k\}$.

The received complex samples $\{r_m\}$, sampled at rate *I/T*, are expressed as

$$r_m = \sum_{k=0}^{M-1} a_k h(mT/I - kT) + n(mT/I), \text{ for } m = 0,1,2,...(MI-1) \tag{4}$$

where $h(t)$ is the channel's impulse response (including transmit filtering), and $\{n(mT/I\}$ are samples of additive noise, assumed to be uncorrelated, have zero mean, and variance σ^2. Because of the presence of the cyclic prefix, the data symbols $\{a_k\}$ can be assumed to be periodic ($a_k = a_{k\pm LM}$, for any integer *L*), as can the impulse response samples ($\{h(mT/I) = h((m/I \pm LM)T)$).

In the discrete frequency domain, (4) becomes

$$R_\ell = H_\ell A_\ell + V_\ell \tag{5}$$

where, for $\ell = 0, 1, 2, ...(MI-1)$,

$$H_\ell = \sum_{m=0}^{MI-1} h(mT/I)\exp(-j2p\frac{m\ell}{MI}) \tag{6a}$$

$$A_\ell = \sum_{m=0}^{M-1} a_m \exp(-j2p\frac{m\ell}{M}) \tag{6b}$$

$$\text{and } V_\ell = \sum_{m=0}^{MI-1} n(mT/I)\exp(-j2p\frac{m\ell}{MI}) \tag{6c}$$

The associated autocorrelation sequences are, from the above correlation assumptions,

$$E(A_{\ell_1} A^*_{\ell_2}) = Md(\ell_1 - \ell_2)\text{mod}\, M,$$

$$E(V_{\ell_1} V^*_{\ell_2}) = MIs^2 d(\ell_1 - \ell_2)$$

$$\text{and } E(R_{\ell_1} R^*_{\ell_2}) = MH_{\ell_1} H^*_{\ell_2} d(\ell_1 - \ell_2)\text{mod}\, M + MIs^2 d(\ell_1 - \ell_2) \tag{7}$$

where $\delta(\ell)$ is the Kronecker delta function, and $0 \le .\ell_1, \ell_2 \le (MI\text{-}1)$.

From (3), (6) and (7), the mean squared error can be expressed as, for m=0, 1, ..(M-1),

$$E(|e_m|^2) = \frac{1}{MI}\sum_{\ell_1=0}^{MI-1}\sum_{\ell_2=0}^{MI-1} W_{\ell_1} W^*_{\ell_2}\left[H_{\ell_1} H^*_{\ell_2} d(\ell_1 - \ell_2)_{\text{mod}\, M} + s^2 d(\ell_1 - \ell_2)\right]$$

$$-\frac{2}{MI}\text{Re}\{\sum_{\ell=0}^{MI-1} W_\ell H_\ell [1 + \sum_{k\in F_B} f_k \exp(j2p\frac{k\ell}{M})]\} + 1 + \sum_{k\in F_B} |f_k|^2 + 1 \tag{8a}$$

which can also be expressed as

$$E(\left|e_m{}^2\right|) = \frac{1}{MI}\sum_{\ell=0}^{M-1}\left|\mathbf{W}_\ell{}'\mathbf{H}_\ell - F_\ell\right|^2 + \frac{s^2}{MI}\sum_{\ell=0}^{M-1}\left|\mathbf{W}_\ell\right|^2, \tag{8b}$$

$$\text{where } \mathbf{W}_\ell = \begin{bmatrix} W_\ell \\ W_{\ell+M} \\ . \\ W_{\ell+(I-1)M} \end{bmatrix}, \quad \mathbf{H}_\ell = \begin{bmatrix} H_\ell \\ H_{\ell+M} \\ . \\ H_{\ell+(I-1)M} \end{bmatrix}, \quad \text{and } F_\ell = 1 + \sum_{k\in F_B} f_k^* \exp(-j2p\frac{k\ell}{M}) \tag{8c}$$

Superscript prime means transpose of a vector or matrix. Minimizing this with respect to the coefficients $\{W_\ell\}$ by setting the derivatives to zero results in a set of linear equations for $\{ W_\ell, W_{\ell+M}, \ldots W_{\ell+(I-1)M}, \}$:

$$\mathbf{W}_\ell = [s^2\mathbf{I} + \mathbf{H}_\ell^*\mathbf{H}_\ell']^{-1}\mathbf{H}_\ell^* F_\ell \tag{9}$$

This solution for the optimum frequency domain forward filter coefficients can be expressed as [Qur85], [Cla98], for ℓ=0, 1, 2, …(MI-1),

$$W_\ell = \frac{H_\ell^*[1 + \sum_{k\in F_B} f_k^* \exp(-j2p\frac{k\ell}{M})]}{s^2 + \left|\hat{H}_\ell\right|^2} \;. \tag{10}$$

where we have defined, for $\ell = 0,1,2,..(M-1)$,

$$\left|\hat{H}_\ell\right|^2 = \sum_{k=0}^{I-1}\left|H_{(\ell+kM)\bmod MI}\right|^2.$$

The mean squared error, minimized with respect to the forward filter coefficients, can then be expressed as

$$\text{MMSE} = \frac{s^2}{MI}\sum_{\ell=0}^{M-1}\frac{\left|F_\ell\right|^2}{s^2 + \left|\hat{H}_\ell\right|^2}. \tag{11}$$

Now minimizing (11) with respect to the feedback coefficients $\{f_k, k \hat{I} F_B,\}$ in F_ℓ, we get a set of B linear equations in the optimum $\{f_k, k \hat{I} F_B,\}$ that minimize the mean squared error, which can be expressed in the following matrix form:

$$\mathbf{Vf} = -\mathbf{v} \qquad (12a)$$

$$\text{where } \mathbf{f} = (f_{k_1}, f_{k_2}, ... f_{k_B})', \qquad (12b)$$

$$\mathbf{v} = (v_{k_1}, v_{k_2}, ... v_{k_B})', \qquad (12c)$$

$$\mathbf{V} = \begin{bmatrix} v_0 & v_{k_1-k_2} & . & v_{k_1-k_B} \\ v_{k_2-k_1} & v_0 & v_{k_2-k_3} & . \\ . & . & . & . \\ v_{k_B-k_1} & . & . & v_0 \end{bmatrix}, \qquad (12d)$$

$$\text{and } v_k = \frac{s^2}{M} \sum_{\ell=0}^{M-1} \frac{\exp(-j2p\frac{\ell k}{M})}{s^2 + |\hat{H}_\ell|^2} \;. \qquad (12e)$$

To minimize complexity, B, the number of feedback coefficients, should be as low as possible. This can be accomplished if the indices k_i Î F_B correspond to the relative estimated delays of the largest channel impulse response echoes. In most of our later examples, B=1 and F_B has just one non-zero index – the relative delay of the largest echo. B=0 corresponds to linear equalization.

6. Equalizer Training Using Least Squares Minimization

We consider the estimation of the equalizer parameters $\{W_\ell\}$ and $\{f_k\}$, from the reception of N consecutive training blocks, each consisting of a sequence of P known transmitted training symbols $\{a_k, k=0, 1, ..P-1\}$. With no loss of generality, we assume that the same training sequence is transmitted in every training block. The length of a training block, P, may be equal to or less than the length of a data block M and it is preceded by a cyclic prefix. If it is less than M, P is picked to be at least equal to the maximum expected channel impulse response length in data symbol intervals. If $P<M$, the forward filter parameters derived from training, $\{\tilde{W}_\ell,\ \ell = 0,1,..PI\}$ can be *interpolated* to values to be used for blocks of M as shown later.

Estimation of the equalizer parameters is based on least-squares (LS) optimization. The mth sample of the nth training block can be expressed as

$$e_m^{(n)} = \frac{1}{P}\sum_{\ell=0}^{P-1}\left[\frac{1}{I}\sum_{k=0}^{I-1}\tilde{W}_{\ell+kP}R_{\ell+kP}^{(n)}\right]\exp(j\frac{2p}{P}\ell m) - \sum_{k\in F_B}\tilde{f}_k^* a_{m-k} - a_m,$$

$$\text{where } m = 0, 1, 2, ...(P-1),\ n = 1, 2, ..N. \qquad (13)$$

Defining, for ℓ=0, 1, 2,…(P-1),

$$\tilde{\mathbf{W}}_\ell = \left(\tilde{W}_\ell \quad \tilde{W}_{\ell+P} \quad . \quad \tilde{W}_{\ell+(I-1)P}\right)' \qquad (14a)$$

$$\text{and } \mathbf{R}_\ell^{(n)} = \frac{1}{I}\left(R_\ell^{(n)} \quad R_{\ell+P}^{(n)} \quad . \quad R_{\ell+(I-1)P}^{(n)}\right)' \qquad (14b)$$

$$\text{and the operation } \langle X \rangle = \frac{1}{N}\sum_{n=1}^{N} X^{(n)}, \qquad (14c)$$

and transforming into the frequency domain, we can write the sum of squared errors, which is to be minimized with respect to the equalizer coefficients as

$$\sum_{m=0}^{P-1} \left\langle \left| e_m^{(n)} \right|^2 \right\rangle$$

$$= \frac{1}{P} \sum_{\ell=0}^{P-1} \left| A_\ell \right|^2 \tilde{\mathbf{W}}_\ell^H \tilde{\mathbf{U}}_\ell \tilde{\mathbf{W}}_\ell - \frac{2}{P} \mathrm{Re}\left[\sum_{\ell=0}^{P-1} \left| A_\ell \right|^2 \tilde{\mathbf{W}}_\ell^H \tilde{\mathbf{H}}_\ell^* F_\ell \right] + \frac{1}{P} \sum_{\ell=0}^{P-1} \left| A_\ell \right|^2 \left| \tilde{F}_\ell \right|^2, \tag{16}$$

where superscript H stands for complex conjugate transpose, and

$$\tilde{\mathbf{U}}_\ell = \left\langle \frac{\mathbf{R}_\ell \mathbf{R}_\ell^H}{\left| A_\ell \right|^2} \right\rangle, \tilde{\mathbf{H}}_\ell = \left\langle \frac{\mathbf{R}_\ell}{A_\ell} \right\rangle, \quad \text{and } \tilde{F}_\ell = 1 + \sum_{k \in F_B} \tilde{f}_k^* \exp(-j2p \frac{\ell k}{P}). \tag{17}$$

Minimizing (16) with respect to $\tilde{\mathbf{W}}_\ell$ and $\{\tilde{f}_k\}$ we get

$$\tilde{\mathbf{W}}_\ell = (\tilde{\mathbf{U}}_\ell)^{-1} \tilde{\mathbf{H}}_\ell^* \tilde{F}_\ell \text{ for } \ell = 0,1,2,...(P-1), \tag{18}$$

and

$$\tilde{\mathbf{V}} \tilde{\mathbf{f}} = -\tilde{\mathbf{v}} \tag{19a}$$

where $\tilde{\mathbf{f}} = (\tilde{f}_{k_1}, \tilde{f}_{k_2}, ... \tilde{f}_{k_B})'$, (19b)

$$\tilde{\mathbf{v}} = (\tilde{v}_{k_1}, \tilde{v}_{k_2}, ... \tilde{v}_{k_B})', \tag{19c}$$

$$\tilde{\mathbf{V}} = \begin{bmatrix} \tilde{v}_0 & \tilde{v}_{k_1-k_2} & . & \tilde{v}_{k_1-k_B} \\ \tilde{v}_{k_2-k_1} & \tilde{v}_0 & \tilde{v}_{k_2-k_3} & . \\ . & . & . & . \\ \tilde{v}_{k_B-k_1} & . & . & \tilde{v}_0 \end{bmatrix}, \tag{19d}$$

and $\tilde{v}_k = \sum_{\ell=0}^{P-1} \left| A_\ell \right|^2 \left[1 - \tilde{\mathbf{H}}_\ell^H \tilde{\mathbf{U}}_\ell^{-1} \tilde{\mathbf{H}}_\ell \right] \exp(-j2p \frac{\ell k}{P})$, (19e)

Equations (17)-(19) summarize the least squares equalizer training algorithm. For the special case of I=1 sample per data symbol, vectors and matrices become scalars, and

$$\tilde{U}_\ell = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{R_\ell^{(n)}}{A_\ell}\right|^2, \quad \tilde{H}_\ell = \frac{1}{N}\sum_{n=1}^{N}\frac{R_\ell^{(n)}}{A_\ell}, \tag{20a}$$

$$\text{and } \tilde{W}_\ell = \frac{\tilde{H}_\ell^*}{\tilde{U}_\ell}\left[1+\sum_{k\in F_B}\tilde{f}_k^*\exp(-j2p\,\frac{\ell k}{P})\right] \tag{20b}$$

$$\text{Also, } \tilde{v}_k = \sum_{\ell=0}^{P-1}\left|A_\ell\right|^2\left[1-\frac{\left|\tilde{H}_\ell\right|^2}{\tilde{U}_\ell}\right]\exp(-j2p\,\frac{\ell k}{P}) \tag{20c}$$

It is clear that $\tilde{H}_\ell$ is an estimate of H_ℓ, and that $\tilde{U}_\ell$ estimates $s^2+\left|H_\ell\right|^2$.

Interpolation from P forward equalizer coefficients to M coefficients is done in the frequency domain: the inverse FFT, of length P, of each component of the vector $\frac{\mathbf{R}_\ell^{(n)}}{A_\ell}$ is computed, the resulting sequences are padded with zeroes to length M, and the FFT is taken; the resulting version of $\frac{\mathbf{R}_\ell^{(n)}}{A_\ell}$ is of length M, and is used to compute $\tilde{\mathbf{H}}_\ell$ and $\tilde{\mathbf{U}}_\ell$.

Alternative methods of training could be based on LMS or RLS algorithms [Cla98]. For example, iterating estimates from the nth to the $(n+1)$th training block can be described by

$$\tilde{\mathbf{W}}_\ell^{(n+1)} = \tilde{\mathbf{W}}_\ell^{(n)} - mE_\ell^{(n)}\mathbf{R}_\ell^{(n)*} \text{ for } \ell = 0,1,\ldots P-1 \tag{21a}$$

$$\text{where } E_\ell^{(n)} = \sum_{m=1}^{P-1} e_m^{(n)}\exp(-j\,\frac{2p\ell m}{P}) \tag{21b}$$

$$f_k^{(n+1)} = f_k^{(n)} + m\sum_{m=1}^{P-1} e_m^{(n)*}a_{m-k}^{(n)} \quad \text{for } k\in F_B \tag{21c}$$

Those methods can also be used for tracking: i.e. updating estimates from block to block, using either training sequences or the receiver's decisions on the previous data block.

The sequence of P transmitted training symbols $\{a_k, k=0, 1, ..P-1\}$ is known as a *unique word*. Its length is chosen to be at least the maximum expected channel delay spread, measured in symbol intervals. Ideally, its DFT $\{A_\ell$ for $\ell=0,1,\ldots P-1\}$ should have equal, or nearly equal magnitude for all indices ℓ; the corresponding cyclic autocorrelation function of $\{a_k, k=0, 1, ..P-1\}$ should ideally be zero for non-zero lags. Such an ideal training sequence ensures that each frequency component of the channel is probed uniformly to provide the estimates of $\{H_\ell\}$ and of $\{|H_\ell|^2+\sigma^2\}$ implicit in the quantities $\{H_\ell\}$ and $\{U_\ell\}$ respectively. For unique word lengths P which are powers of two, such as 64 or 256, polyphase Frank-Zadoff sequences [Fra62] or Chu sequences [Chu72] are suitable. If binary-valued sequences are more desirable from a hardware implementation standpoint, length 2^n-1, pn sequences can be modified by adding a small dc value in quadrature, as suggested by Milewski [Mil83]. Each of these types of sequences have the desired property that $|A_\ell|$ is a constant value.

Consider two or more back-to-back unique words, periodically inserted within a data sequence, for synchronization and equalizer training purposes. The first of these back-to-back unique words acts as a cyclic prefix, and absorbs any intersymbol interference from previous data. The second and subsequent words form a periodic sequence that have the ideal periodic autocorrelation property described above. An example of the aforesaid description, which may be found in a continuous downstream transmission, is shown in Fig. 6. Note that the payload data blocks of Fig. 6 are preceded and followed by contiguous unique words.

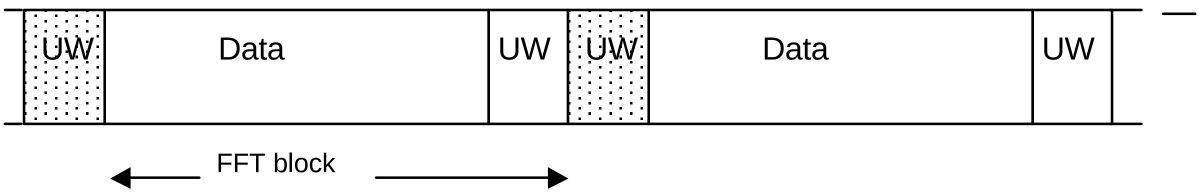


Fig. 6 Continuous downstream transmission showing unique words used for training shaded.

The unique words immediately preceding the data (indicated by shading in Fig. 6) form a periodic sequence of training blocks. Each unique word following the data segment serves as a cyclic prefix for the succeeding unique word and also shields it from intersymbol interference from non-training data. The overhead fraction is $2P/(D+2P)$, if the unique word length is P, and the data block length is D. An example of bursty uplink or downlink transmission is shown in Fig. 7. Here, an extra unique word is added as a cyclic prefix, and the total overhead fraction is $3P/(D+3P)$.

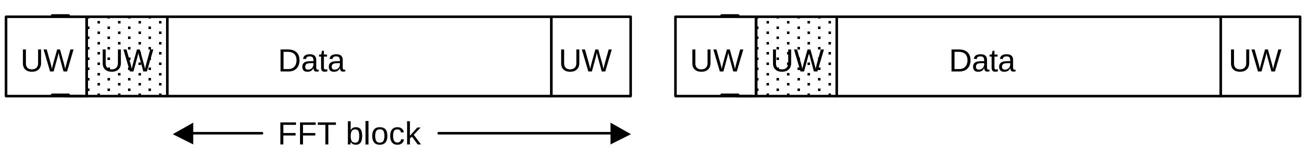


Fig. 7 Bursty transmission with unique words for training shaded

The use of unique words for equalizer training, as well as interpolation in the frequency domain is the counterpart of the use of pilot tones and frequency domain interpolation in OFDM systems. As an alternative to the approach in Section 6, estimation of the frequency domain equalizer coefficients can also be done from the unique words in the time domain – crosscorrelating the received unique word segments with unique words in order to estimate the channel impulse response, followed by transformation to the frequency domain and use of equations (10) – (12). In this case, a separate estimate of the noise variance σ^2 is also necessary.

7. Performance

7.1 Ideal Performance

The bit error rate performance of SC-FDE, using perfect channel knowledge, and with training, has been evaluated by simulation using several models of broadband wireless channels with multipath fading. Fig. 8 depicts the delay spread profile for channel 'SUI-5', one of six channel models adopted by IEEE 802.16.3 for evaluating broadband wireless systems in 2-11 GHz bands [Erc01].

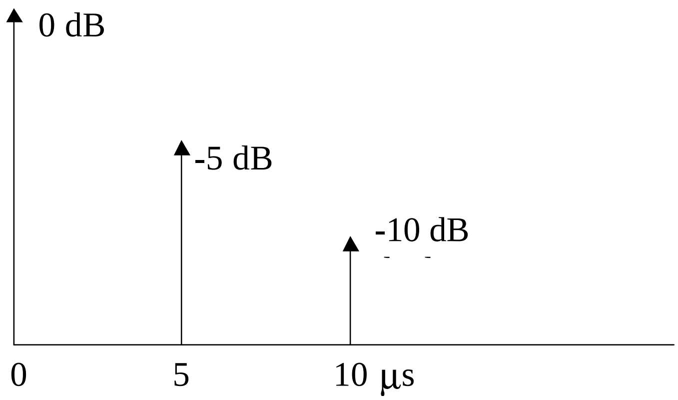


Fig. 8. SUI-5 delay spread profile

This is a high delay spread model associated with the use of omnidirectional antennas in suburban hilly environments. The channel has a maximum delay spread of 10 μs., and a rms delay spread of 3.05 μs. Each of the three echoes at 0, 5 and 10 μs. is modelled as an independent complex gaussian random variable, with relative variances of 0, -5 and -10 dB respectively. The fading was modeled as quasi-static (unchanging during a FFT block).

QPSK, 16QAM and 64QAM single carrier and OFDM systems were simulated against this model for a range of received signal to noise ratios, each with 20,000 random channel realizations. For each channel realization, obtained by Monte Carlo simulation, the BER was computed, and then the BER was, in turn, averaged over all channel realizations. BER results were compiled for interleaved convolutionally coded systems with various code rates. Code rates greater than 1/2 were realized by optimally puncturing [Yas84] a standard rate 1/2, constraint length 7 code with generator polynomials (133,171). The coded bits were interleaved and mapped into transmitted M-ary QAM data symbols using Gray mapping. The resulting coding scheme is known as “bit-interleaved coded modulation (BICM)”, which has been shown [Li99] to perform within a dB or two of trellis coded modulation (TCM) over additive white Gaussian noise channels and to outperform TCM over fast fading channels where the interleaver spans multiple fades. (Therefore, for a coded OFDM block transmitted over a frequency selective fading (multipath) channel, which resembles a fast fading temporal channel, BICM is the most suitable coding scheme [Cai98]).

Each FFT block consists of 512 QAM symbols. Row-column interleaving was used within each FFT block, where the data bits are written by row and mapped to QAM symbols by column, each row consisting of 32 bits. The raised cosine roll-off factor used for the single carrier systems was 10%.

Fig. 9 shows the average bit error probability evaluated over a range of average signal to noise ratios using a rate 1/2 code and for the three (4-16-64) QAM constellations, for the following system configurations:

- Single carrier modulation using frequency domain linear equalization (FD-LE)
- OFDM, based on optimum linear equalization and optimally weighted soft decision MLSE decoding
- Single carrier modulation using ideal frequency domain decision feedback equalization (FD-DFE), assuming an infinite-length feedback filter and correct feedback (no decision errors)
- For an upper bound comparison, the matched filter bound (MFB) (performance with a matched filter receiver and no intersymbol interference).

In these simulations, perfect channel and output SNR estimation was assumed for all systems.

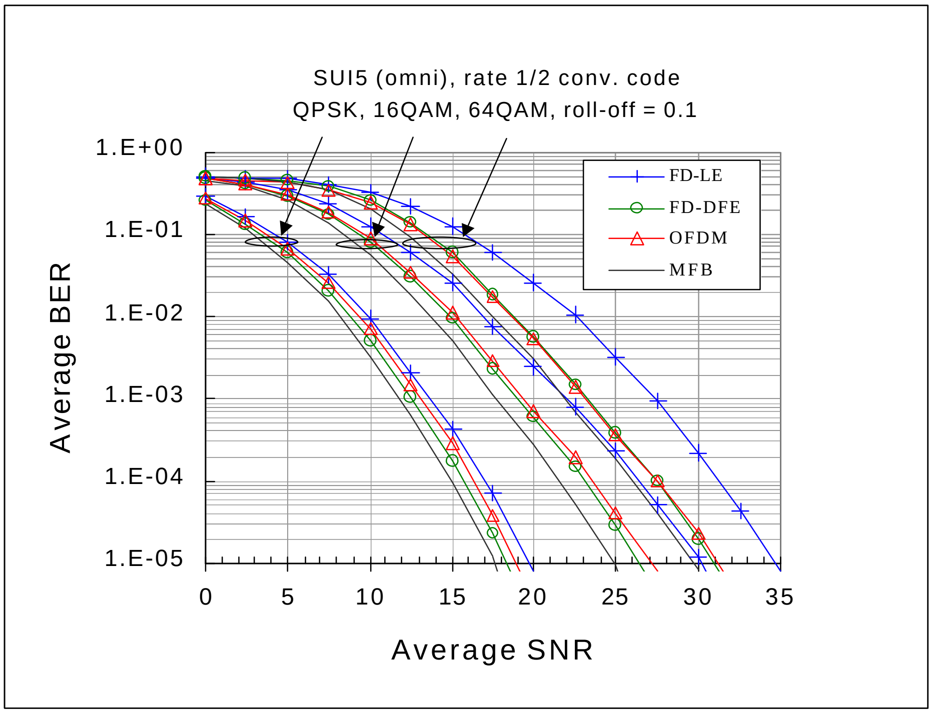


Fig. 9. Performance comparison for systems with perfect channel knowledge on SUI-5 channel for code rate 1/2

The results of Fig. 9 suggest that for a channel operating at lower average SNR's -- where QPSK modulation is appropriate, OFDM, FD-LE and ideal FD-DFE SC systems all perform within about 1.5-2 dB of one another. The ideal FD-DFE performs to within about 1 dB of the ideal matched filter bound for QPSK. OFDM performed slightly better than FD-LE, and slightly worse than the ideal FD-DFE. For 16QAM and 64QAM, there is a somewhat larger spread among the systems, but with the same relative rankings. In particular, the spread between FD-LE and other systems become larger for higher level modulation.

Both OFDM and FD-LE suffer from noise enhancement in severe frequency selective Rayleigh fading channels, such as SUI-5, but their corresponding decoders operate and perform somewhat differently. For the FD-LE, the noise enhancement loss increases with the average input SNR, i.e., when the channel has deep nulls and the SNR is high (typically required for high level modulation), the linear equalizer will try harder to invert the nulls and, as a result, the noise in those null locations is also amplified. In contrast, OFDM can exploit the *independent* (Rayleigh-distributed) *known* (from channel estimation measurements) gain and phase of each subchannel and combine the useful energy across all subchannels through coding and interleaving. However, the performance of OFDM in frequency selective fading is sensitive to the code rate (and strength of the code) used.

Figs. 10 (a)-(c) shows the peformance over the SUI-5 channel using QPSK and higher code rate (2/3. 3/4, and 7/8). Note that for rate 3/4 and 7/8, OFDM actually performs worse than the FD-LE. For an uncoded system, our simulations and those of others [Sar94], [Czy97], [Kad97] show that the BER performance of OFDM is far inferior to that of the linear and DFE single carrier systems, since without coding, the Rayleigh fading on each OFDM subchannel presents the appearance of flat Rayleigh fading to the OFDM symbol detector, even when the multipath channel taps themselves do not fade.

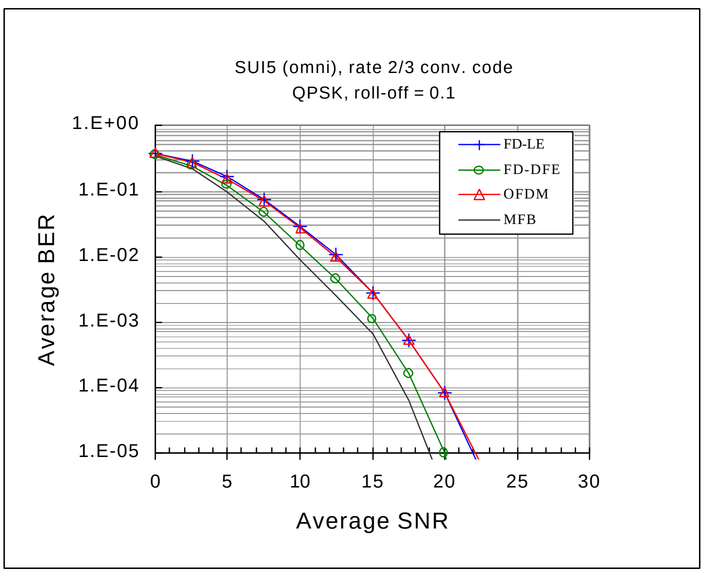


Fig. 10a Performance comparison for systems with perfect channel knowledge on SUI-5 channel model for QPSK with code rate 2/3

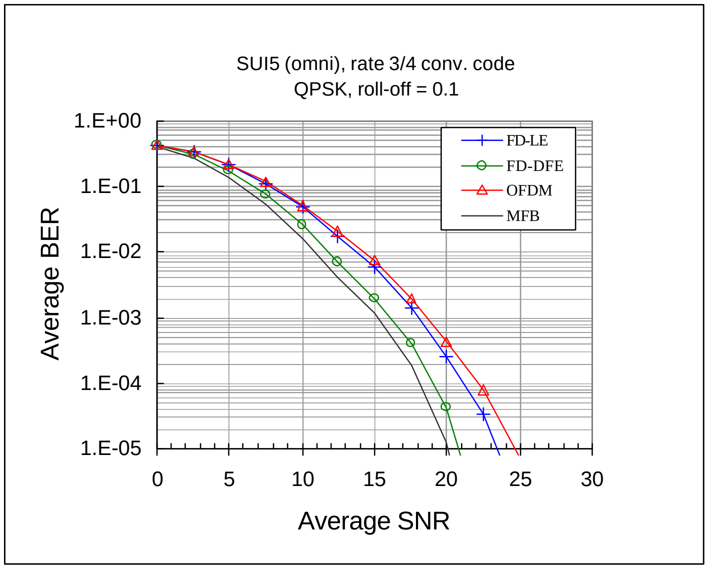


Fig. 10b Performance comparison for systems with perfect channel knowledge on SUI-5 channel model for QPSK with code rate 3/4

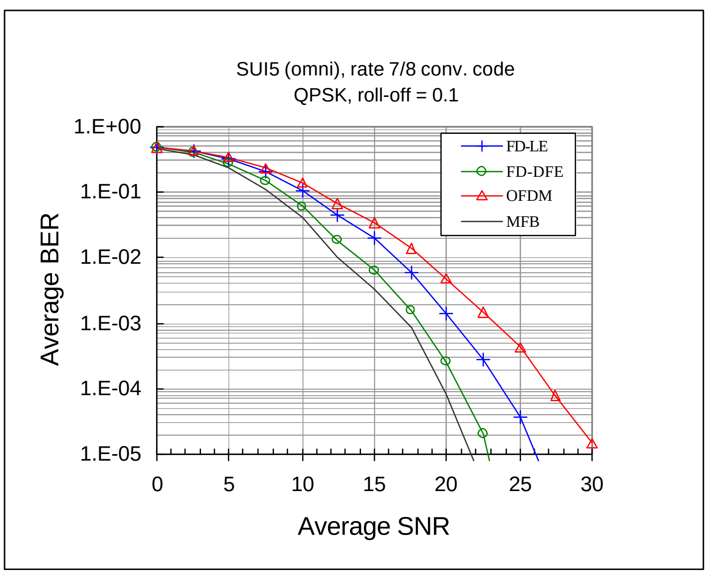


Fig. 10c Performance comparison for systems with perfect channel knowledge on SUI-5 channel model for QPSK with code rate 7/8

These results seem to agree with other measures of performance found in the literature. For example,[Aue98] compared coded linearly equalized single-carrier modulation (FD-LFE) with coded OFDM over a two-ray Rayleigh fading channel, using a the computational cutoff rate for each scheme as its measure of performance.. Both schemes had similar performance at low to moderate code rates, while the coded linearly equalized single-carrier modulation system exhibited better performance at higher code rates.

7.2 Performance with Equalizer Training and Finite DFE

Fig. 11 shows the simulated performance of a single carrier frequency domain equalizer, with training, on the SUI-5 channel. 16-QAM symbols with rate ¾ BICM are used in the data payload. *N*=4 Frank-Zadoff sequence unique word training blocks, each of length 64, are used to estimate the forward and feedback coefficients of linear equalizers and DFE's, the latter with only one feedback tap. The channel estimation procedure is that described in Section 6, and uses frequency domain interpolation, described in Section 6 to extrapolate from 64-training symbol FFT blocks to the 1024-symbol FFT blocks that are used for frequency domain equalization in this example. The time delay of the feedback tap is set to the delay of the largest-magnitude delayed echo in the channel impulse response. If more feedback taps were to be used, their delays would be set to the second-largest, third-largest etc. echo delays. In practice these delays could be estimated during the training period from the inverse FFT of $\tilde{\mathbf{H}}_\ell$ in (17).

The performance measure used in Fig. 11 is the probability (over an ensemble of 20,000 SUI-5 channel realizations) that the bit error rate of the rate 3/4, 16QAM coded system is worse than10^{-6}. Call this probability (that a minimal BER is not maintained) an ‘outage probability’. In these and all further examples, such outage probabilities are computed by an analytic method. This method involves assessing the equalizer output SNR for each channel realization, using a lookup table to map the SNR to an outage probability, and then averaging the outage probabilities over all channel realizations. The outage probability vs. outage probability table used in this procedure comes from previously tabulated AWGN BER curves for a rate ¾ BICM code using 16-QAM.

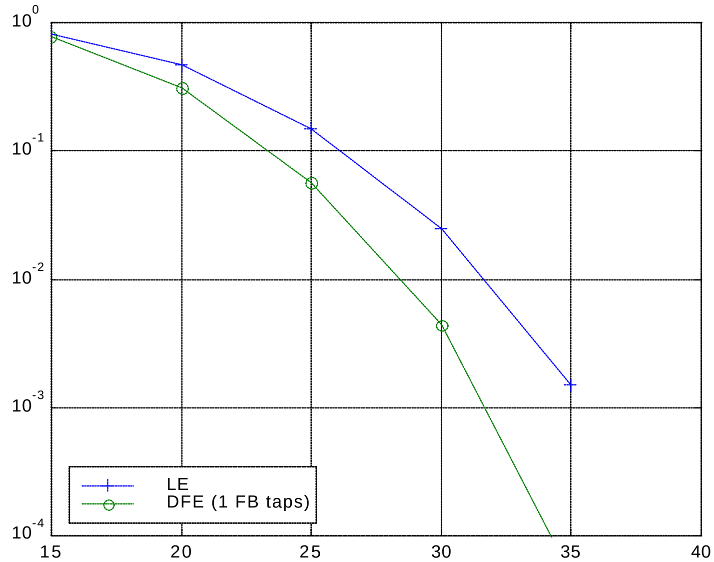


Fig. 11. Outage probability performance for SC- frequency domain equalization for SUI-5 Rayleigh fading channel. Training over 4 64-symbol training blocks

Fig. 11 demonstrates that the relatively simple 1-feedback tap DFE betters the linear equalizer performance at a 10^{-3} outage level by about 2 to 3 dB. Fig. 11 also shows that the performance loss relative to perfect channel estimation is on the order of 1 to 1.5 dB. In this simulation, the effect of decision error propagation in the DFE is not included. A separate investigation of error propagation in the 1-tap DFE revealed an average bit error rate increase corresponding to a degradation of about 1 dB or less.

Frequency domain linear and decision feedback equalization were also simulated for a channel with an exponential delay spread profile with an average rms delay spread of 1 µs.It is worth noting that measurements reported in [Erc99] suggested that multipath in non line of sight suburban environments is reasonably well modelled as such with a 99% worst case rms delay spread of about 0.3 µs. Again, 20,000 random channel realizations were simulated, each with 16 Rayleigh-fading taps at 0.1 µs. intervals. The transmitted signal was 16 QAM with a rate 3/4 BICM convolutional code. Fig. 12a and 12b show the BER>10^{-6} outage probability for perfect channel estimation and 4-block training, respectively. Linear equalization and 1-tap DFE performance is shown, and Fig. 12a also shows the matched filter bound performance.

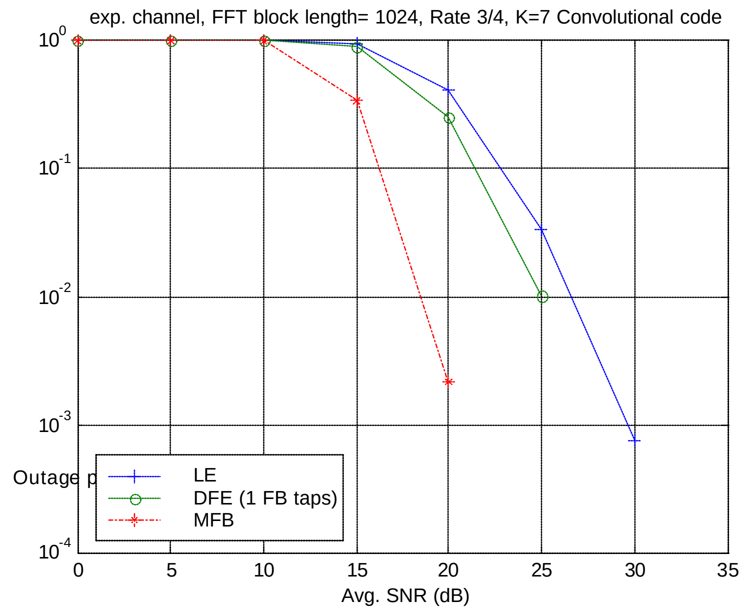


Fig. 12a Outage probability performance for SC- frequency domain equalization and matched filter bound for an exponential delay profile Rayleigh fading channel with rms delay spread of 1 µs. Perfect channel estimation.

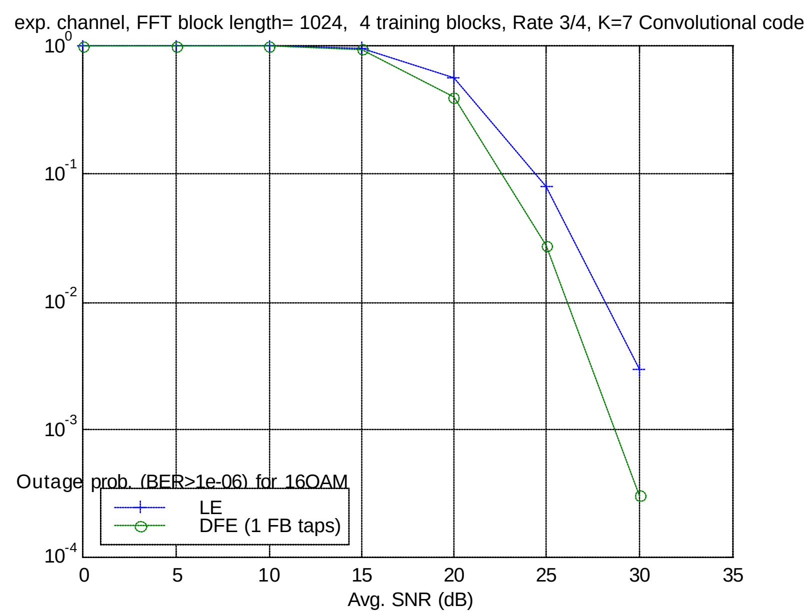


Fig. 12b Outage probability performance for SC- frequency domain equalization for an exponential delay profile Rayleigh fading channel with rms delay spread of 1 µs. Training over 4 64-symbol training blocks

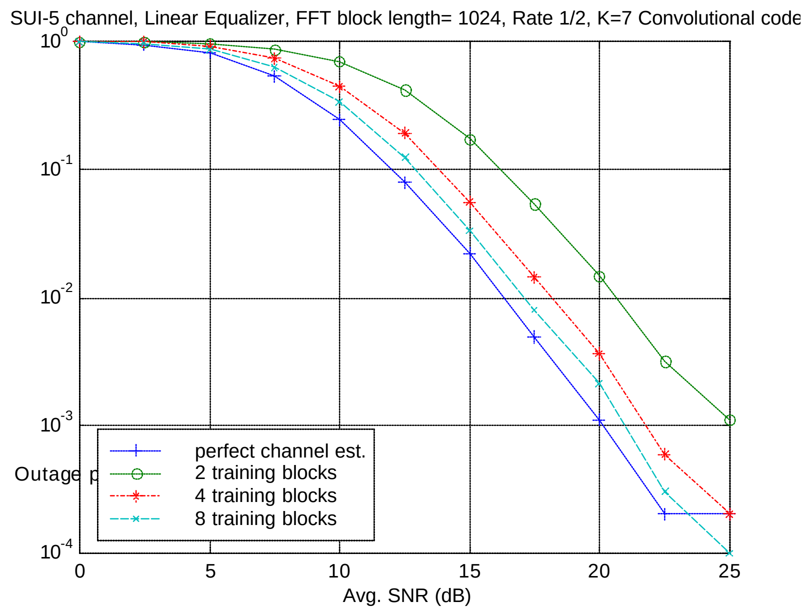


Fig. 13a Outage performance for QPSK, rate ½ code, for different numbers of 64-symbol training blocks

Figs. 13a and 13b show the outage performance for QPSK with a rate ½ code and 64QAM with rate ¾ code, respectively, on the SUI-5 channel for different numbers of 64-symbol training blocks, as well as for perfect channel estimation. The number of training blocks to achieve a performance degradation of about 2 dB is 4,

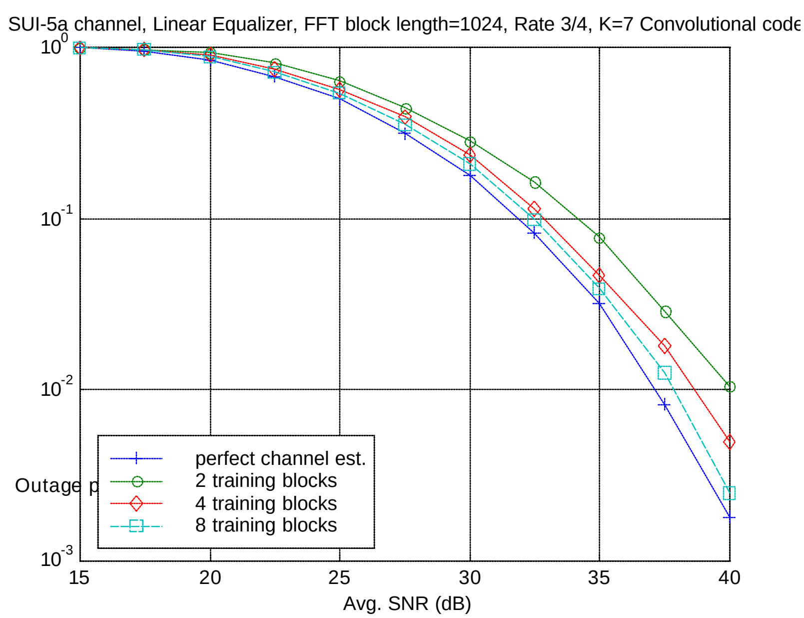


Fig. 13b Outage performance for 64QAM, rate 3/4 code, for different numbers of 64-symbol training blocks

while 8 training blocks come within 1 dB or less of perfect channel estimation performance. In all cases, the channel is assumed unknown before the use of the training symbols to estimate the channel. What's more, only the training symbols are used to estimate the channel: no decision direction of payload symbols is used to progressively improve the accuracy of the channel estimates.

Note that 8 training blocks, each of length 64 symbols, represents a total of one half the length of a single FFT block (1024 symbols) in this example. In a burst environment, one or more of these FFT blocks may compose a single burst.

The relative performance of DFE's with different numbers of feedback taps is shown in Fig. 14, for 64QAM, rate ¾ code on SUI-5. Also shown for comparison is the matched filter bound. In these, results, perfect channel estimation and correct decision feedback were assumed.

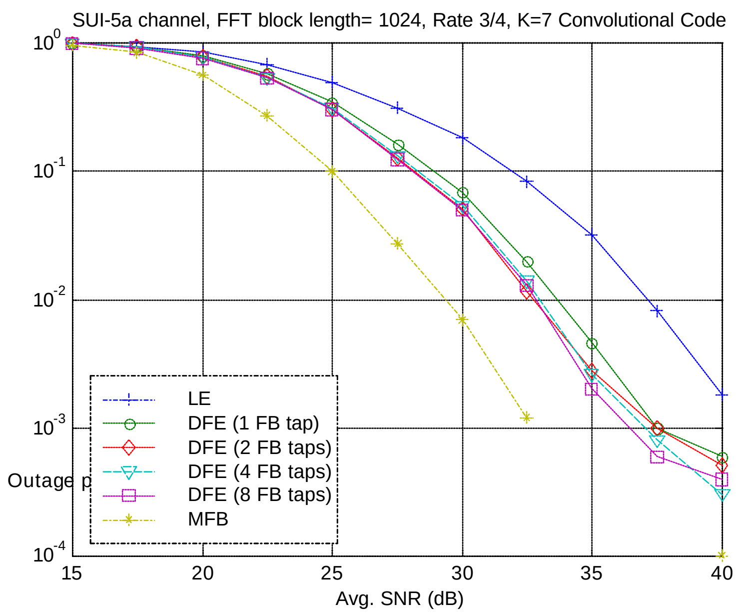


Fig. 14 Effect of different numbers of feedback taps

8. Summary

Fixed wireless access systems providing high bit rate access to residential and small-business subscribers in non-LOS environments may be subject to severe time dispersion, spanning many bit intervals. For such environments, modulation and equalization strategies based on frequency domain processing should be considered to achieve adequate anti-multipath performance with reasonable complexity. Frequency domain processing is the basis for OFDM, and it also applies equally well to single carrier modulation. However, OFDM is very sensitive to power amplifier nonlinearities and frequency offsets. Single-carrier modulation, using linear frequency domain equalization at the receiver (SC-FDE), has less sensitivity to transmitter nonlinearities and to phase noise than does OFDM, its complexity and its performance are similar to those of OFDM. Furthermore the performance of SC-FDE is enhanced when it is combined with simple sparse time-domain decision feedback equalization. Equations (1) and (2) in Section 5, and (14), (17), (18) and (19) in Section 6, specify the signal processing operations carried out for SC-FDE with decision feedback equalization, including training.

Also, as we have seen, single carrier and OFDM systems can potentially coexist for mutual benefit and cost reduction, because of the obvious similarities in their basis frequency domain signal processing functions.

Furthermore, single carrier techniques can be easily combined with MIMO (multiple-input, multiple-output) techniques, in which both transmitting and receiving ends use arrays of antenna elements; MIMO techniques can potentially achieve enormous spectral efficiencies (bit/s/Hz), limited only by the number of diversity antenna elements which can be practically implemented [Fos99]. This, in turn, relieves the delay spread issues, since the desired bit rate is achieved without increasing the symbol rate.

9. References

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